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THE UTILIZATION OF DATA MEASUREMENT RESIDUALS FOR ADAPTIVE KALMAN FILTERING

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Newport, Rhode Island

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PREFACE

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In recent years, the Kalman filter has been utilized extensively for passive target motion analysis (TMA) — an application in which filter divergence is a common problem. Available methods for eliminating divergence ultimately involve increasing filter sensitivity by discounting the influence of past data. However, this procedure makes the filter more susceptible to random errors; therefore, to avoid unnecessary sacrificing of noise performance, adaptive control is required. In this report, the Kalman filter equations are derived and the associated data measurement residuals are examined to determine their suitability for providing adaptive

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20. ABSTRACT (Cont'd)

control. An important relationship between the system performance index and the data residuals is established. By exploiting this relationship, pertinent statistical properties of the performance index are deduced and utilized as a basis for formulating practical adaptive control criteria. A simulation example is presented to demonstrate divergence (e.g., tracking of a maneuvering target) and significant improvement in performance is noted when adaptive control is appended.

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THE UTILIZATION OF DATA MEASUREMENT RESIDUALS FOR ADAPTIVE KALMAN FILTERING

INTRODUCTION

In recent years, the Kalman filter and other recursive estimation schemes have been utilized extensively for target motion analysis (TMA). Unfortunately, in these applications, a common problem known as filter divergence is often encountered. Divergence occurs when the calculated error covariance becomes inconsistent with the actual error covariance. Although there are many possible causes for divergence, a common source is system modelling errors. This is particularly true in TMA work where practical limitations preclude "exact" modelling of the actual target dynamics.

Various methods have been suggested to prevent the growth of modelling errors; fixed memory filters, fading memory filters, and/or injecting artificial plant noise into the system are but a few. A common feature of these methods is that they all attempt to prevent divergence by discounting the influence of past data. While this effectively increases filter response, it also makes the filter more susceptible to random errors. Consequently, to prevent unnecessary sacrificing of noise performance, adaptive control is required.

The need for adaptively controlled filters has provided impetus for the development of numerous "modified" Kalman filtering schemes.[1-4] In all these schemes the adaptive control mechanism is a function of the Kalman filter data measurement residuals. This might be expected since data residuals provide the only consistently reliable basis for assessing solution quality and detecting the onset of divergence. However, it is unfortunate that the importance and practical utility of residuals, particularly in the context of adaptive filtering, have not been adequately stressed. In fact, there is a paucity of technical literature devoted to this subject.

This report analyzes the Kalman filter and its associated data measurement residuals. By utilizing the classical method of least squares, a derivation of the well known filter equations is presented. Although this particular approach is somewhat unorthodox, it is nevertheless appealing since ancillary mathematical formulas that are later needed for analyzing the residuals emerge naturally. Following this, an important relationship between the system performance index and the data measurement residuals is developed and then used to demonstrate the suitability of residuals for providing adaptive control and on-line assessment of solution quality. In addition, pertinent statistical properties of the residuals and performance index are deduced and utilized to establish a basis for formulating practical adaptive control criteria. Finally, a simulation example which demonstrates filter divergence (e.g., tracking of a maneuvering target) is presented.

DERIVATION OF THE KALMAN FILTER EQUATIONS

DESCRIPTION OF THE SYSTEM

Consider a physical system which can be described mathematically by a set of linear stochastic vector difference equations of the form

$$\underline{X}(k+1) = A(k+1,k)\underline{X}(k) + B(k)\underline{W}(k), \tag{1-a}$$

$$\underline{Z}(k+1) = H(k+1)\underline{X}(k+1) + \underline{V}(k+1), \qquad k = 0,1,2,3,\ldots, \tag{1-b}$$

where

- $\underline{X}(k)$ - p-dimensional vector which statistically describes the system states at time T_k,
- $A(k+1,k)$ - $(p \times p)$ deterministic one-step transition matrix for the system,
- $B(k)$ - $(p \times p)$ deterministic matrix of known form,
- $\underline{W}(k)$ - q-dimensional vector of stochastic inputs,
- $\underline{Z}(k+1)$ - m-dimensional vector of noisy data measurements taken at time T_{k+1},
- $H(k+1)$ - $(m \times p)$ deterministic measurement matrix,
- $\underline{V}(k+1)$ - m-dimensional vector of additive measurement noise.

The pertinent statistics associated with this system are as follows:

$$E\{\underline{X}(0)\} = \underline{X}(0,0) \tag{2-a}$$

$$E\{\underline{W}(k)\} = \underline{W}(k,0) \tag{2-b}$$

$$E\{\underline{V}(k)\} = 0 \tag{2-c}$$

$$E\{[\underline{X}(0) - \underline{X}(0,0)][\underline{X}(0) - \underline{X}(0,0]'\} = P(0) \tag{2-d}$$

$$E\{[\underline{W}(j) - \underline{W}(j,0)][\underline{W}(k) - \underline{W}(k,0)]'\} = \delta_{jk}Q(k) \tag{2-e}$$

$$E\{\underline{V}(j)\underline{V}'(k)\} = \delta_{jk}R(k) \tag{2-f}$$

$$E\left\{[\underline{X}(0) - \underline{X}(0,0)][\underline{W}(k) - \underline{W}(k,0)]'\right\} = 0 \tag{2-g}$$

$$E\left\{[\underline{X}(0) - \underline{X}(0,0)]\underline{V}'(k)\right\} = 0 \tag{2-h}$$

$$E\left\{[\underline{W}(j) - \underline{W}(j,0)]\underline{V}'(k)\right\} = 0 \tag{2-i}$$

where

$$\delta_{jk} = \begin{cases} 1 & j = k \\ 0 & j \neq k \end{cases} \tag{2-j}$$

and $E\{\bullet\}$ denotes the statistical expectation operator.

As is well known (e.g., Sage[5]), the formal solution to equation (1-a) is given by

$$\underline{X}(k) = A(k,0)\underline{X}(0) + \sum_{j=0}^{k-1} A(k,j+1)B(j)\underline{W}(j), \qquad k = 1,2,\ldots, \tag{3}$$

which readily reveals the statistical nature of $\underline{X}(k)$ through its functional dependence on the random vectors $\underline{X}(0), \underline{W}(0), \underline{W}(1), \ldots \underline{W}(k-1)$. Consequently, estimates of $\underline{X}(k)$ are all that can be determined. In the absence of additional information it is evident that the optimal estimate of $\underline{X}(k)$ is its a priori mean value, defined by

$$E\left\{\underline{X}(k)\right\} = \underline{X}(k,0) = A(k,0)\underline{X}(0,0) + \sum_{j=0}^{k-1} A(k,j+1)B(j)\underline{W}(j,0). \tag{4}$$

It should be noted that this equation provides an a priori description of expected system behavior. Suppose, however, that further information about the system states is provided in the form of discrete data measurements $\underline{Z}(k)$ which are linearly related to $\underline{X}(k)$ by equation (1-b). If such information is properly utilized, it is reasonable to expect that better estimates of $\underline{X}(k)$ will result. Numerous techniques exist for extracting such estimates from measured data. One of the most powerful is the Kalman filter which will now be derived.

DEVELOPMENT OF THE FILTER EQUATIONS

To analyze the system previously described, assume that n data measurements $\underline{Z}(1), \underline{Z}(2), \ldots \underline{Z}(n)$ are available. The problem at hand is to determine an optimal estimate of the current state vector $\underline{X}(n)$ from these measurements and the a priori information. Here, the criteria for optimality will be defined so that the resulting estimates best fit the measured data while simultaneously minimizing deviations from a priori expected system behavior. For convenience, the following notation will be employed:

$\underline{X}(k, n)$ = optimal estimate of $\underline{X}(k)$ based on n data measurements,

$\underline{W}(k, n)$ = optimal estimate of $\underline{W}(k)$ based on n data measurements.

It can be seen that equation (4), which defines $\underline{X}(k, 0)$, is consistent with this notation since the a priori mean value of $\underline{X}(k)$ represents the optimal estimate of this vector when no data measurements are available. Finally, to ensure that all estimates satisfy the general system equations, the following constraints are imposed:

$$\underline{X}(k+1, n) = A(k+1, k)\underline{X}(k, n) + B(k)\underline{W}(k, n) , \qquad k = 0, 1, 2, \ldots (n-1) . \tag{5}$$

To complete a mathematical formulation of the problem, it is necessary to define a system performance index J(n). To this end, let

$$\begin{aligned} J(n) = {} & 1/2[\underline{X}(0, n) - \underline{X}(0, 0)]' P^{-1}(0)[\underline{X}(0, n) - \underline{X}(0, 0)] \\ & + 1/2 \sum_{l=0}^{n-1} [\underline{W}(l, n) - \underline{W}(l, 0)]' Q^{-1}(l)[\underline{W}(l, n) - \underline{W}(l, 0)] \\ & + 1/2 \sum_{l=1}^{n} [\underline{Z}(l) - H(l)\underline{X}(l, n)]' R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l, n)] . \end{aligned} \tag{6}$$

It might be noted that J(n) is a weighted quality measure of the estimation process based upon the optimality criteria employed. The first two terms in equation (6) account for deviations from expected system behavior, and the last term accounts for cumulative data-fit errors. Because of the way J(n) is defined, it is reasonable to expect that this quantity, or some function thereof, will provide a suitable basis for assessing solution quality on-line. This point is explored further in the next section.

An optimal estimate $\underline{X}(n,n)$ of the current state vector $\underline{X}(n)$ may now be obtained by minimizing $J(n)$ subject to the constraints imposed by equation (5). However, if equation (3) is utilized, these constraints may be rewritten in the equivalent format

$$\underline{X}(k,n) = A(k,0)\underline{X}(0,n) + \sum_{j=0}^{k-1} A(k,j+1)B(j)\underline{W}(j,n), \qquad k = 1,2,\ldots n, \tag{7}$$

which reveals that $J(n)$ is a function of n+1 independent vectors $\underline{X}(0,n), \underline{W}(0,n), \underline{W}(1,n), \ldots \underline{W}(n-1,n)$. Consequently, the minimization of $J(n)$ will require that

$$\frac{\partial J(n)}{\partial \underline{X}(0,n)} = 0\,, \tag{8-a}$$

$$\frac{\partial J(n)}{\partial \underline{W}(k,n)} = 0\,, \qquad k = 0,1,2,\ldots(n-1)\,. \tag{8-b}$$

Performing the operations indicated by equations (8) yields

$$P^{-1}(0)[\underline{X}(0,n) - \underline{X}(0,0)] = \sum_{l=1}^{n} A'(l,0)H'(l)R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)]\,, \tag{9-a}$$

$$Q^{-1}(k)[\underline{W}(k,n) - \underline{W}(k,0)] = \sum_{l=0}^{n-1} \frac{\partial \underline{X}'(l,n)}{\partial \underline{W}(k,n)} H'(l)R^{-1}(l)\,[\underline{Z}(l) - H(l)\underline{X}(l,n)],$$

$$k = 0,1,2,\ldots(n-1)\,. \tag{9-b}$$

By taking the transpose of equation (7), replacing the index k with l, and differentiating the result with respect to $\underline{W}(k,n)$, it follows that

$$\frac{\partial \underline{X}'(l,n)}{\partial \underline{W}(k,n)} = \begin{cases} 0 & l<(k+1) \\ B'(k)A'(l,k+1) & l \geq (k+1) \end{cases}\,. \tag{10}$$

Hence, equation (9-b) reduces to

$$Q^{-1}(k)[\underline{W}(k,n) - \underline{W}(k,0)] = B'(k)\sum_{l=k+1}^{n} A'(l,k+1)H'(l)R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)]\,,$$

$$k = 0,1,2,\ldots(n-1)\,. \tag{11}$$

The determination of $X(n,n)$ first requires that equations (9-a) and (11), or their equivalent, be solved for the unknown vectors $\underline{X}(0,n), \underline{W}(0,n), \underline{W}(1,n), \underline{W}(2,n), \ldots \underline{W}(n-1,n)$. To this end, it proves convenient to introduce an auxiliary vector $\underline{S}(k,n)$ according to the formula

$$B'(k)A'(k,k+1)\underline{S}(k,n) = Q^{-1}(k)[\underline{W}(k,n) - \underline{W}(k,0)], \quad k = 0,1,2,\ldots(n-1). \tag{12}$$

Substituting this expression into equation (11) and utilizing the transition matrix property

$$A'(l,k+1) = A'(k,k+1)A'(l,k) \tag{13}$$

leads to the relationship

$$\underline{S}(k,n) = \sum_{l=k+1}^{n} A'(l,k)H'(l)R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)], \quad k = 0,1,2,\ldots(n-1). \tag{14}$$

If the index k in equation (14) is set equal to zero and the result compared to equation (9-a), the following boundary condition may be derived:

$$\underline{S}(0,n) = P^{-1}(0)[\underline{X}(0,n) - \underline{X}(0,0)]. \tag{15}$$

Similarly, setting $k = (n-1)$ in equation (14) yields

$$\underline{S}(n-1,n) = A'(n,n-1)H'(n)R^{-1}(n)[\underline{Z}(n) - H(n)\underline{X}(n,n)]. \tag{16}$$

Again referring to equation (14), it can be seen that

$$\underline{S}(k+1,n) = \sum_{l=k+2}^{n} A'(l,k+1)H'(l)R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)]$$

$$= A'(k,k+1)\underline{S}(k,n) - H'(k+1)R^{-1}(k+1)[\underline{Z}(k+1) - H(k+1)\underline{X}(k+1,n)],$$

$$k = 0,1,2,\ldots(n-1), \tag{17}$$

where the transition matrix identity

$$A(k,k) = I, \qquad k = 0,1,2,\ldots, \tag{18}$$

has been utilized. Note that for $k = (n-1)$ a new vector $\underline{S}(n,n)$ now appears in equation (17) and is defined by

$$\underline{S}(n,n) = A'(n-1,n)\underline{S}(n-1,n) - H'(n)R^{-1}(n)[\underline{Z}(n) - H(n)\underline{X}(n,n)]. \tag{19}$$

However, substituting equation (16) into (19) and noting that

$$A(l,k) = A^{-1}(k,l) \tag{20}$$

leads to the boundary condition

$$\underline{S}(n,n) = 0 . \tag{21}$$

Equations (7), (12), (15), (17), and (21) comprise a coupled set of linear inhomogeneous difference equations and associated boundary conditions that are mathematically equivalent to equations (9-a) and (11). This set, which is summarized below, may therefore be used to determine $\underline{X}(n,n)$.

$$\underline{X}(k+1,n) = A(k+1,k)\underline{X}(k,n) + B(k)\underline{W}(k,0) + B(k)Q(k)B'(k)A'(k,k+1)\underline{S}(k,n), \tag{22-a}$$

$$\underline{S}(k+1,n) = A'(k,k+1)\underline{S}(k,n) - H'(k+1)R^{-1}(k+1)[\underline{Z}(k+1) - H(k+1)\underline{X}(k+1,n)], \tag{22-b}$$

$$\underline{S}(0,n) = P^{-1}(0)[\underline{X}(0,n) - \underline{X}(0,0)], \tag{22-c}$$

$$\underline{S}(n,n) = 0, \qquad k = 0,1,2,\dots(n-1). \tag{22-d}$$

It is interesting to note that the above equations are similar to the discrete canonical equations derived by Sage[5] using dynamic optimization theory. Both sets describe linear two-point boundary value problems which may be resolved by the method of discrete invariant imbedding.[5] However, an alternate procedure will be employed here, which yields the desired vector $\underline{X}(n,n)$ in a relatively simple fashion. Thus, assume that one additional data measurement is taken at time T_{n+1}. The data measurement sequence is now defined by $\{\underline{Z}(1), \underline{Z}(2), \dots \underline{Z}(n+1)\}$ and equation set (22) is replaced by

$$\underline{X}(k+1,n+1) = A(k+1,k)\underline{X}(k,n+1) + B(k)\underline{W}(k,0) + B(k)Q(k)B'(k)A'(k,k+1)\underline{S}(k,n+1), \tag{23-a}$$

$$\underline{S}(k+1,n+1) = A'(k,k+1)\underline{S}(k,n+1) - H'(k+1)R^{-1}(k+1)[\underline{Z}(k+1) - H(k+1)\underline{X}(k+1,n+1)], \tag{23-b}$$

$$\underline{S}(0,n+1) = P^{-1}(0)[\underline{X}(0,n+1) - \underline{X}(0,0)], \tag{23-c}$$

$$\underline{S}(n+1,n+1) = 0, \qquad k = 0,1,2,\dots n. \tag{23-d}$$

Next, define

$$\Delta\underline{X}(k) = \underline{X}(k, n+1) - \underline{X}(k, n), \tag{24-a}$$

$$\Delta\underline{S}(k) = \underline{S}(k, n+1) - \underline{S}(k, n). \tag{24-b}$$

Subtracting the expressions in equation set (23) from the corresponding expressions in equation set (22) and utilizing equations (24) then yields

$$\Delta\underline{X}(k+1) = A(k+1, k)\Delta\underline{X}(k) + B(k)Q(k)B'(k)A'(k, k+1)\Delta\underline{S}(k), \tag{25-a}$$

$$\Delta\underline{S}(k+1) = A'(k, k+1)\Delta\underline{S}(k) + H'(k+1)R^{-1}(k+1)H(k+1)\Delta\underline{X}(k+1), \tag{25-b}$$

$$\Delta\underline{S}(0) = P^{-1}(0)\Delta\underline{X}(0), \tag{25-c}$$

$$\Delta\underline{S}(n) = \underline{S}(n, n+1). \tag{25-d}$$

Though not immediately evident, the homogeneous equation set (25) is much simpler to solve than the original equation set (22). To illustrate, assume a solution of the form

$$\Delta\underline{X}(k) = P(k)\Delta S(k), \qquad k = 0, 1, 2, \ldots (n), \tag{26}$$

where $P(0)$ is known, and $P(1)$, $P(2), \ldots P(n)$ are matrices to be determined. (Note that a similar type homogeneous solution of the form $\underline{X}(k, n) = P(k)\underline{S}(k, n)$ cannot be applied to equation set (22).) Substituting equation (26) in (25-a) and (25-b), respectively, and utilizing equation (20) then yields

$$P(k+1)\Delta\underline{S}(k+1) = N(k+1)A'(k, k+1)\Delta S(k), \tag{27-a}$$

$$[P^{-1}(k+1) - H'(k+1)R^{-1}(k+1)H(k+1)]P(k+1)\Delta\underline{S}(k+1) = A'(k, k+1)\Delta\underline{S}(k), \tag{27-b}$$

where

$$N(k+1) = A(k+1, k)P(k)A'(k+1, k) + B(k)Q(k)B'(k). \tag{28}$$

In order to satisfy equation set (27) for all allowable values of k, it is necessary that

$$P(k+1) = [N(k+1) + H'(k+1)R^{-1}(k+1)H(k+1)]^{-1}. \tag{29}$$

In addition, equations (24-a), (25-d), and (26) may be combined to produce the expression

$$\underline{X}(n, n+1) = \underline{X}(n, n) + P(n)\underline{S}(n, n+1). \tag{30}$$

Equation (23-a), coupled with equations (28) and (30), then yields the relationship

$$\underline{X}(n+1,n+1) = A(n+1,n)\underline{X}(n,n) + B(n)\underline{W}(n,0) + N(n+1)A'(n,n+1)\underline{S}(n,n+1). \tag{31}$$

Note also that equations (23-b) and (23-d) lead to

$$A'(n,n+1)\underline{S}(n,n+1) = H'(n+1)R^{-1}(n+1)[\underline{Z}(n+1) - H(n+1)\underline{X}(n+1,n+1)]. \tag{32}$$

Finally, substituting equation (32) into equation (31) to eliminate $\underline{S}(n,n+1)$ and performing some algebraic manipulation produce the desired results:

$$\begin{aligned}\underline{X}(n+1,n+1) = \underline{X}(n+1,n) + P(n+1)H'(n+1)R^{-1}(n+1)[\underline{Z}(n+1)\\ - H(n+1)\underline{X}(n+1,n)],\end{aligned} \tag{33-a}$$

$$\underline{X}(n+1,n) = A(n+1,n)\underline{X}(n,n) + B(n)\underline{W}(n,0)\ . \tag{33-b}$$

Equations (28), (29), and (33) provide a mathematical procedure for optimally estimating the current state of a system from measured data and a priori statistical information. As expected, these equations are equivalent to the Kalman filtering formulas and can be put into the standard format by utilizing the following matrix identities:

$$\begin{aligned}G(n+1) &= N(n+1)H'(n+1)[H(n+1)N(n+1)H'(n+1) + R(n+1)]^{-1}\\ &= P(n+1)H'(n+1)R^{-1}(n+1),\end{aligned} \tag{34-a}$$

$$\begin{aligned}P(n+1) &= [N^{-1}(n+1) + H'(n+1)R^{-1}(n+1)H(n+1)]^{-1}\\ &= [I - G(n+1)H(n+1)]N(n+1).\end{aligned} \tag{34-b}$$

For completeness, the Kalman filter equations are summarized on the following page in algorithm form.

Kalman Filter Equations

$\underline{X}(0,0)$ - initial estimate of state-vector

$P(0)$ - initial estimate of state-vector covariance matrix

$$\underline{X}(n+1,n) = A(n+1,n)\underline{X}(n,n) + B(n)\underline{W}(n,0)\,, \tag{35-a}$$

$$N(n+1) = A(n+1,n)P(n)A'(n+1,n) + B(n)Q(n)B'(n)\,, \tag{35-b}$$

$$G(n+1) = N(n+1)H'(n+1)[H(n+1)N(n+1)H'(n+1) + R(n+1)]^{-1}\,, \tag{35-c}$$

$$\underline{X}(n+1,n+1) = \underline{X}(n+1,n) + G(n+1)[\underline{Z}(n+1) - H(n+1)\underline{X}(n+1,n)]\,, \tag{35-d}$$

$$P(n+1) = [I - G(n+1)H(n+1)]N(n+1)\,, \quad n = 0,1,2,\ldots. \tag{35-e}$$

ANALYSIS OF THE KALMAN FILTER DATA MEASUREMENT RESIDUALS

REPRESENTATION OF THE PERFORMANCE INDEX IN TERMS OF RESIDUALS

Data measurement residuals, which arise naturally during computations performed by the Kalman filter, are defined by the equation

$$\underline{Y}(k) = \underline{Z}(k) - H(k)\underline{X}(k, k-1), \qquad k = 1, 2, \ldots. \tag{36}$$

In effect, $\underline{Y}(k)$ is a measure of the error between the actual data measurement vector at time T_k and the best available prediction of that vector. When taken collectively, these residuals provide reliable indication of how well the state-vector estimates "fit" the measured data. In turn, such information can be effectively utilized to assess solution quality. However, it is important to recognize that since the residuals are random quantities they only provide information of a statistical nature. The magnitude of any one residual has little significance by itself.

In the preceding section, it was pointed out that the performance index $J(n)$ provides a basis for assessing solution quality on-line. Since the residuals may also be used for this purpose, it would appear that these quantities are related to $J(n)$ in some way. To determine this relationship, first note that equations (6) and (9-a) may be combined to yield the expression

$$\begin{aligned} J(n) = 1/2 \sum_{l=0}^{n-1} & [\underline{W}(l, n) - \underline{W}(l, 0)]' Q^{-1}(l)[\underline{W}(l, n) - \underline{W}(l, 0)] \\ & + 1/2 \sum_{l=1}^{n} [\underline{Z}(l) - H(l)\underline{X}(l, n) + H(l)A(l, 0)\underline{X}(0, n) \\ & - H(l)A(l, 0)\underline{X}(0, 0)]' R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l, n)]. \end{aligned} \tag{37}$$

However, equations (4) and (7) reveal that

$$\begin{aligned} & \underline{X}(k, n) - A(k, 0)\underline{X}(0, n) \pm A(k, 0)\underline{X}(0, 0) \\ & \qquad = \underline{X}(k, 0) + \sum_{j=0}^{k-1} A(k, j+1)B(j)[\underline{W}(j, n) - \underline{W}(j, 0)]. \end{aligned} \tag{38}$$

Consequently, $J(n)$ may be rewritten in the form

$$J(n) = 1/2 \sum_{l=0}^{n-1} [\underline{W}(l,n) - \underline{W}(l,0)]' Q^{-1}(l) [\underline{W}(l,n) - \underline{W}(l,0)]$$

$$+ 1/2 \sum_{l=1}^{n} \sum_{j=0}^{l-1} [\underline{W}(j,n) - \underline{W}(j,0)]' B'(j) A'(l,j+1) H'(l) R^{-1}(l) [\underline{Z}(l) - H(l)\underline{X}(l,n)]$$

$$+ 1/2 \sum_{l=1}^{n} [\underline{Z}(l) - H(l)\underline{X}(l,0)]' R^{-1}(l) [\underline{Z}(l) - H(l)\underline{X}(l,n)] . \tag{39}$$

The second term on the right-hand side of equation (39) can be simplified by noting that

$$\sum_{l=1}^{n} \sum_{j=0}^{l-1} [\underline{W}(j,n) - \underline{W}(j,0)]' B'(j) A'(l,j+1) H'(l) R^{-1}(l) [\underline{Z}(l) - H(l)\underline{X}(l,n)]$$

$$= \sum_{l=0}^{n-1} [\underline{W}(l,n) - \underline{W}(l,0)]' B'(l) \sum_{j=l+1}^{n} A'(j,l+1) H'(j) R^{-1}(j) [\underline{Z}(j) - H(j)\underline{X}(j,n)]$$

$$= \sum_{l=0}^{n-1} [\underline{W}(l,n) - \underline{W}(l,0)]' Q^{-1}(l) [\underline{W}(l,n) - \underline{W}(l,0)] , \tag{40}$$

where the final result derives from equation (11). If equation (40) is now substituted into equation (39) it follows that

$$J(n) = 1/2 \sum_{l=1}^{n} [\underline{Z}(l) - H(l)\, \underline{X}(l,0)]' R^{-1}(l) [\underline{Z}(l) - H(l)\underline{X}(l,n)] . \tag{41}$$

Next, note that equations (35-a), (35-d), and (36) may be combined to produce the expression

$$\underline{X}(k+1,k+1) = A(k+1,k)\underline{X}(k,k) + B(k)\underline{W}(k,0) + G(k+1)\underline{Y}(k+1) , \tag{42}$$

which can also be written in the equivalent form

$$\underline{X}(k,k) = A(k,0)\underline{X}(0,0) + \sum_{j=0}^{k-1} A(k,j+1)B(j)\underline{W}(j,0) + \sum_{j=1}^{k} A(k,j)G(j)\underline{Y}(j) . \tag{43}$$

Comparing this equation with equation (4) reveals that

$$\underline{X}(k,0) = \underline{X}(k,k) - \sum_{j=0}^{k} A(k,j)G(j)\underline{Y}(j) . \tag{44}$$

Finally, substituting equation (35-d) into equation (44) to eliminate $\underline{X}(k,k)$ yields

$$\underline{X}(k,0) = \underline{X}(k,k-1) + G(k)\underline{Y}(k) - \sum_{j=1}^{k} A(k,j)G(j)\underline{Y}(j) , \tag{45}$$

from which it is readily deduced that

$$\underline{Z}(k) - H(k)\underline{X}(k,0) = [I - H(k)G(k)]\underline{Y}(k) + H(k) \sum_{j=1}^{k} A(k,j)G(j)\underline{Y}(j) . \tag{46}$$

If equation (46) is now substituted into equation (41), the performance index $J(n)$ takes the form

$$J(n) = 1/2 \sum_{l=1}^{n} \underline{Y}'(l)[I - G'(l)H'(l)]R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)]$$

$$+ 1/2 \sum_{l=1}^{n} \sum_{j=1}^{l} \underline{Y}'(j)G'(j)A'(l,j)H'(l)R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)] . \tag{47}$$

This cumbersome expression can be greatly simplified by noting that

$$\sum_{l=1}^{n} \sum_{j=1}^{l} \underline{Y}'(j)G'(j)A'(l,j)H'(l)R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)]$$

$$= \sum_{l=1}^{n} \underline{Y}'(l)G'(l) \sum_{j=l}^{n} A'(j,l)H'(j)R^{-1}(j)[\underline{Z}(j) - H(j)\underline{X}(j,n)]$$

$$= \sum_{l=1}^{n} \underline{Y}'(l)G'(l)\{H'(l)R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)] + \underline{S}(l,n)\} , \tag{48}$$

where the final results derive from equations (14) and (18). Hence, by combining equations (47) and (43), the expression for $J(n)$ reduces to

$$J(n) = 1/2 \sum_{l=1}^{n} \underline{Y}'(l)\left\{R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n)] + G'(l)\underline{S}(l,n)\right\}. \tag{49}$$

However, since n is an arbitrary integer, it follows immediately that

$$J(n+1) = 1/2 \sum_{l=1}^{n+1} \underline{Y}'(l)\left\{R^{-1}(l)[\underline{Z}(l) - H(l)\underline{X}(l,n+1)] + G'(l)\underline{S}(l,n+1)\right\}. \tag{50}$$

Subtracting equation (49) from equation (50) then yields the recursive formula

$$J(n+1) - J(n) = 1/2 \sum_{l=1}^{n} \underline{Y}'(l)[G'(l)\Delta\underline{S}(l) - R^{-1}(l)H(l)\Delta\underline{X}(l)]$$

$$+ 1/2\underline{Y}'(n+1)\left\{R^{-1}(n+1)[\underline{Z}(n+1) - H(n+1)\underline{X}(n+1,n+1)] + G'(n+1)S(n+1,n+1)\right\}, \tag{51}$$

where $\Delta\underline{X}(l)$ and $\Delta\underline{S}(l)$ are defined by equation set (24). The next step in the reduction process involves substituting equations (25-d) and (26) into equation (51) to obtain

$$J(n+1) - J(n) = 1/2 \sum_{l=1}^{n} \underline{Y}'(l)[G'(l) - R^{-1}(l)H(l)P(l)]\Delta\underline{S}(l)$$

$$+ 1/2\underline{Y}'(n+1)R^{-1}(n+1)[\underline{Z}(n+1) - H(n+1)\underline{X}(n+1,n+1)]. \tag{52}$$

Note, however, that $P(l)$ and $R(l)$ are both symmetric matrices; consequently, the identity

$$G'(k) - R^{-1}(k)H(k)P(k) = 0, \qquad k = 0,1,2,\ldots, \tag{53}$$

is easily derived by taking the matrix transpose of equation (34-a). This relationship shows that the series appearing in equation (51) vanishes. In addition, the last remaining term on the right-hand side of equation (51) can be simplified by utilizing equations (35-c), (35-d), (35-e), and (36). The result is

$$\underline{Y}'(n+1)R^{-1}(n+1)[\underline{Z}(n+1) - H(n+1)X(n+1,n+1)]$$

$$= \underline{Y}'(n+1)R^{-1}(n+1)[I - H(n+1)G(n+1)]\underline{Y}(n+1)$$

$$= \underline{Y}'(n+1)[H(n+1)N(n+1)H'(n+1) + R(n+1)]^{-1}\underline{Y}(n+1). \tag{54}$$

The preceding equations may now be combined to produce a simple recursive formula for $J(n)$; that is

$$J(n+1) = J(n) + 1/2\underline{Y}'(n+1)[H(n+1)N(n+1)H'(n+1) + R(n+1)]^{-1}\underline{Y}(n+1)\,, \qquad n = 0,1,2,\ldots. \tag{55}$$

An initial condition for $J(n)$ can also be deduced from equation (49). More precisely, since

$$\begin{aligned} J(1) &= 1/2\underline{Y}'(1)\{R^{-1}(1)[\underline{Z}(1) - H(1)\underline{X}(1,1)] + G(1)\underline{S}(1,1)\} \\ &= 1/2\underline{Y}'(1)R^{-1}(1)[\underline{Z}(1) - H(1)\underline{X}(1,1)] \\ &= 1/2\underline{Y}'(1)R^{-1}(1)[I - H(1)G(1)]\underline{Y}(1) \\ &= 1/2\underline{Y}'(1)[H(1)N(1)H'(1) + R(1)]^{-1}\underline{Y}(1)\,, \end{aligned}$$

it follows from equation (55) that

$$J(0) = 0\,. \tag{56}$$

Although equations (55) and (56) are convenient for numerical work, a closed form representation of $J(n)$ is often more desirable for analytical purposes. Such a representation takes the form

$$J(n) = 1/2\sum_{l=1}^{n} \underline{Y}'(l)[H(l)N(l)H'(l) + R(l)]^{-1}\underline{Y}(l)\,. \tag{57}$$

A comparison of this expression with equation (6) shows that the data measurement residuals also provide a weighted quality measure of the estimation process based upon the optimality criteria employed. As such, it now becomes evident why these residuals can be utilized effectively to assess filter performance.

An important feature of equation (57) not to be overlooked is that all quantities needed to calculate $J(n)$ can be extracted directly from the Kalman filter equations without further computation. In addition, once the terms in the series have been evaluated they need not be recomputed with each new data measurement. Consequently, for practical applications, the utilization of equation (57) — in lieu of equation (6) — can result in a significant saving of computer time.

STATISTICAL PROPERTIES OF THE RESIDUALS

Earlier in the discussion, it was noted that the data measurement residuals are random variables and can only provide information of a statistical nature. The performance index also possesses these characteristics. Consequently, knowledge of the statistical properties of both $J(n)$ and $Y(k)$ is required before meaningful information can be extracted from these quantities.

The mean value of $Y(k)$ can be determined rather easily by combining equations (1-b) and (46) to obtain

$$H(k)[\underline{X}(k) - \underline{X}(k,0)] + \underline{V}(k) = [I - H(k)G(k)]\underline{Y}(k) + H(k)\sum_{j=1}^{k} A(k,j)G(j)\underline{Y}(j) . \quad (58)$$

Taking the expected value of this equation and utilizing equations (2-c) and (4) then yields

$$[I - H(k)G(k)]E\left\{\underline{Y}(k)\right\} + H(k)\sum_{j=1}^{k} A(k,j)G(j)E\left\{\underline{Y}(j)\right\} = 0 . \quad (59)$$

Since the index k in equation (59) is arbitrary, it can easily be shown by induction that

$$E\left\{\underline{Y}(k)\right\} = 0 , \qquad k = 1,2,\ldots . \quad (60)$$

In order to determine the covariance of $\underline{Y}(k)$, it is first necessary to prove the following identity:

$$E\left\{[\underline{X}(k) - \underline{X}(k,k-1)][\underline{X}(k) - \underline{X}(k,k-1)]'\right\} = N(k) . \quad (61)$$

To this end, assume that equation (61) is true for some fixed value of k. By combining equations (1-a), (1-b), (35-a), and (35-d) it can also be shown that

$$\begin{aligned}\underline{X}(k+1) - \underline{X}(k+1,k) &= A(k+1,k)[\underline{X}(k) - \underline{X}(k,k)] + B(k)[\underline{W}(k) - \underline{W}(k,0)] \\ &= A(k+1,k)[\underline{X}(k) - \underline{X}(k,k-1) - G(k)\underline{Z}(k) + G(k)H(k)\underline{X}(k,k-1)] \\ &\quad + B(k)[\underline{W}(k) - \underline{W}(k,0)] \\ &= A(k+1,k)[I - G(k)H(k)][\underline{X}(k) - \underline{X}(k,k-1)] \\ &\quad - A(k+1,k)G(k)\underline{V}(k) + B(k)[\underline{W}(k) - \underline{W}(k,0)] . \end{aligned} \quad (62)$$

Consequently, the statistical properties depicted in equation set (2) and equation (61) can be utilized, together with equation (62), to produce the expression

$$E\left\{[\underline{X}(k+1) - \underline{X}(k+1,k)][\underline{X}(k+1) - \underline{X}(k+1,k)]'\right\}$$

$$= A(k+1,k)\left\{[I - G(k)H(k)]N(k)[I - G(k)H(k)]' + G(k)R(k)G'(k)\right\}A'(k+1,k)$$

$$+ B(k)Q(k)B'(k) \; . \qquad (63)$$

Note, however, that

$$P(k) = [I - G(k)H(k)]N(k)$$

$$= [I - G(k)H(k)]N(k)[I - G(k)H(k)]' + G(k)R(k)G'(k) \; . \qquad (64)$$

Consequently, equation (63) reduces to

$$E\left\{[\underline{X}(k+1) - \underline{X}(k+1,k)][\underline{X}(k+1) - \underline{X}(k+1,k)]'\right\}$$

$$= A(k+1,k)P(k)A'(k+1,k) + B(k)Q(k)B'(k) = N(k+1) \, , \qquad (65)$$

where the final result derives from equation (35-b). It has therefore been shown that if equation (61) is valid for any value of k, then so is equation (65). The last remaining step in this induction proof is to show that equation (61) is valid for some value of k. In particular, let $k = 1$. For this case, equations (1-a) and (35-a) can be combined to yield

$$\underline{X}(1) - \underline{X}(1,0) = A(1,0)[\underline{X}(0) - \underline{X}(0,0)] + B(0)[\underline{W}(0) - \underline{W}(0,0)] \; . \qquad (66)$$

If equation set (2) is again utilized, together with equations (35-b) and (54), it then follows that

$$E\left\{[\underline{X}(1) - \underline{X}(1,0)][\underline{X}(1) - \underline{X}(1,0)]'\right\}$$

$$= A(1,0)P(0)A'(1,0) + B(0)Q(0)B'(0) = N(1) \, , \qquad (67)$$

and the validity of equation (61) is established.

The covariance of $\underline{Y}(k)$ can now be determined in a simple manner by recalling that

$$\underline{Y}(k) = H(k)[\underline{X}(k) - \underline{X}(k,k-1)] + \underline{V}(k) \ . \tag{68}$$

As such, the desired result

$$E\{\underline{Y}(k)\underline{Y}'(k)\} = H(k)N(k)H'(k) + R(k) \tag{69}$$

can be deduced immediately from equation set (2) and equation (61). It might also be noted that

$$\begin{aligned} &E\{\underline{Y}'(k)[H(k)N(k)H'(k) + R(k)]^{-1}\underline{Y}(k)\} \\ &= \text{Trace}\{[H(k)N(k)H'(k) + R(k)]^{-1}E\{\underline{Y}(k)\underline{Y}'(k)\}\} \\ &= \text{Trace}\{[H(k)N(k)H'(k) + R(k)]^{-1}[H(k)N(k)H'(k) + R(k)]\} \\ &= \text{Trace}\{I\} = m \ , \end{aligned} \tag{70}$$

where m is the dimension of the measurement vector $\underline{Z}(k)$. With the aid of equation (70), the mean value of the performance index can be easily determined as follows:

$$\begin{aligned} E\{J(n)\} &= 1/2 \sum_{l=1}^{n} E\{\underline{Y}'(l)[H(l)N(l)H'(l) + R(l)]^{-1}\underline{Y}(l)\} \\ &= 1/2 \sum_{l=1}^{n} m = (1/2)mn \ . \end{aligned} \tag{71}$$

A final statistic of importance is the variance of J(n). In general, a closed form expression for this quantity cannot readily be obtained since fourth order statistical moments must be computed. However, if all statistics are Gaussian, then J(n) will be Chi-Square distributed[6] with $m \cdot n$ degrees of freedom. Under these conditions, the variance of J(n) is given by

$$E\{[J(n) - E\{J(n)\}]^2\} = (1/2)mn \ . \tag{72}$$

It is important to note that this statistic provides a measure of the permissible deviation in $J(n)$ from its expected value. Unfortunately, since larger and larger deviations are permitted as the number of data measurements increases, it becomes progressively more difficult to extract reliable information from $J(n)$. More precisely, after many measurements have been processed, large deviations in $J(n)$ cannot be attributed solely to filter divergence, but instead may result simply from the accumulation of random errors. The concomitant ambiguity makes it impossible to accurately assess the true filter status.

Fortunately, there are various ways to circumvent this difficulty. The simplest technique is to introduce a "modified" performance index of the form

$$L(n) = 1/2 \sum_{l=1}^{n} \gamma^{n-l} \left\{ \underline{Y}'(l)[H(l)N(l)H'(l) + R(l)]^{-1}\underline{Y}(l) - m \right\} , \tag{73}$$

where γ is a weighting factor which satisfies the inequality

$$0 < \gamma < 1 . \tag{74}$$

The mean and variance of $L(n)$ are given by

$$E\left\{L(n)\right\} = 0 , \tag{75-a}$$

$$E\left\{L^2(n)\right\} = \frac{m}{2}\left(\frac{1-\gamma^{2n}}{1-\gamma^2}\right) . \tag{75-b}$$

Note that these statistics remain bounded as the number of data measurements increase. In fact

$$\lim_{n\to\infty} E\left\{L^2(n)\right\} = \frac{m}{2(1-\gamma^2)} . \tag{76}$$

For finite values of n, any desired variance in the range

$$\frac{m}{2} < E\left\{L^2(n)\right\} < \frac{mn}{2} \tag{77}$$

can be obtained by choosing γ appropriately. This flexibility is achieved by exponentially discounting past filter performance so that $L(n)$ is influenced much more by current events than is $J(n)$. As such, $L(n)$ will provide a re-

liable quantitative measure of current solution quality which can be effectively utilized for adaptive control. For example, if the values of $L(n)$ are statistically consistent, then it can be safely assumed that the Kalman filter is performing satisfactorily. However, if $L(n)$ becomes statistically inconsistent during the course of operation (e.g., $L(n)$ continually exceeds its one sigma limit), this would indicate that the filter is diverging, and appropriate preventative action should be taken. It might be noted that the effectiveness of any divergence prevention scheme can also be easily assessed by continually monitoring $L(n)$ for statistical consistency.

For convenience, a recursive algorithm for numerically computing $L(n)$ as well as a summary of its statistical properties is presented below:

$$L(0) = 0 , \tag{78-a}$$

$$\underline{Y}(n+1) = \underline{Z}(n+1) - H(n+1)\underline{X}(n+1,n) , \tag{78-b}$$

$$L(n+1) = \gamma L(n) + 1/2\left\{\underline{Y}'(n+1)[H(n+1)N(n+1)H'(n+1) + R(n+1)]^{-1}\underline{Y}(n+1) - m\right\}, \tag{78-c}$$

$$E\left\{\underline{Y}(n+1)\right\} = 0 , \tag{78-d}$$

$$E\left\{\underline{Y}(n+1)\underline{Y}'(n+1)\right\} = H(n+1)N(n+1)H'(n+1) + R(n+1) , \tag{78-e}$$

$$E\left\{L(n+1)\right\} = 0 , \tag{78-f}$$

$$E\left\{L^2(n+1)\right\} = \frac{m}{2}\left(\frac{1-\gamma^{2n+2}}{1-\gamma^2}\right), \qquad 0<\gamma<1 , \tag{78-g}$$

where m is the dimension of the data measurement vector, and $n = 0,1,2,\ldots$.

SIMULATION RESULTS

To demonstrate the advantages of using a modified performance index for adaptive control, a target motion analysis (TMA) experiment was conducted. Since target maneuvers are a primary cause of divergence in present TMA systems, the geometry depicted in figure 1 was chosen. In this geometry, target maneuvers consisting of 180 degree turns occur at approximately 6 minutes and 12 minutes. Ownship motion is as shown and the turning rate of both vehicles is 3° per second.* Ownship speed is 18 knots and target speed is 20 knots. Two experiments were performed with this geometry.

First, an extended non-adaptive Kalman filter-TMA algorithm which utilizes a constant target acceleration plant description was employed. Data measurements consisted of bearing and range rate information. Figures 2, 3, and 4 are plots of the target parameter estimate errors versus time. In figures 5 and 6, normalized values† of the performance index $J(n)$, defined by equation (57), and the modified performance index $L(n)$, defined by equation (73), are presented. From examination of figures 2, 3, and 4, it is evident that the estimation errors become unacceptably large after the first target maneuver. In addition, deviations (from expected behavior) apparent in figures 5 and 6 indicate that the filter statistics are no longer consistent with their a priori expected values. Clearly, the target maneuvers have caused the Kalman filter to diverge.

In the second experiment, an extended adaptive Kalman filter-TMA algorithm which also utilizes a constant target acceleration plant description was employed. Here, however, plant noise was injected into the system to accommodate mis-modelling errors introduced by target maneuvers. The plant noise covariance matrix is adaptively varied according to the behavior of the modified performance index. Figures 7, 8, and 9 are plots of the target parameter estimate errors versus time for this case. Figures 10 and 11 are the respective analogues to figures 5 and 6. Inspection of figures 7, 8, and 9 reveals that the error divergence observed in figures 2, 3, and 4 is no longer present; and examination of figures 10 and 11 indicates that the adaptive control mechanism has effectively realigned the filter statistics with their a priori expected values. Further examination of figures 10 and 11 also reveals the sluggish nature of the performance index $J(n)$ and the more responsive behavior of the modified performance index $L(n)$.

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*Turning radius of ownship is not shown in figure 1.

†In this simulation, the performance indices $J(n)$ and $L(n)$ have been normalized so that both have expected values of unity for all n.

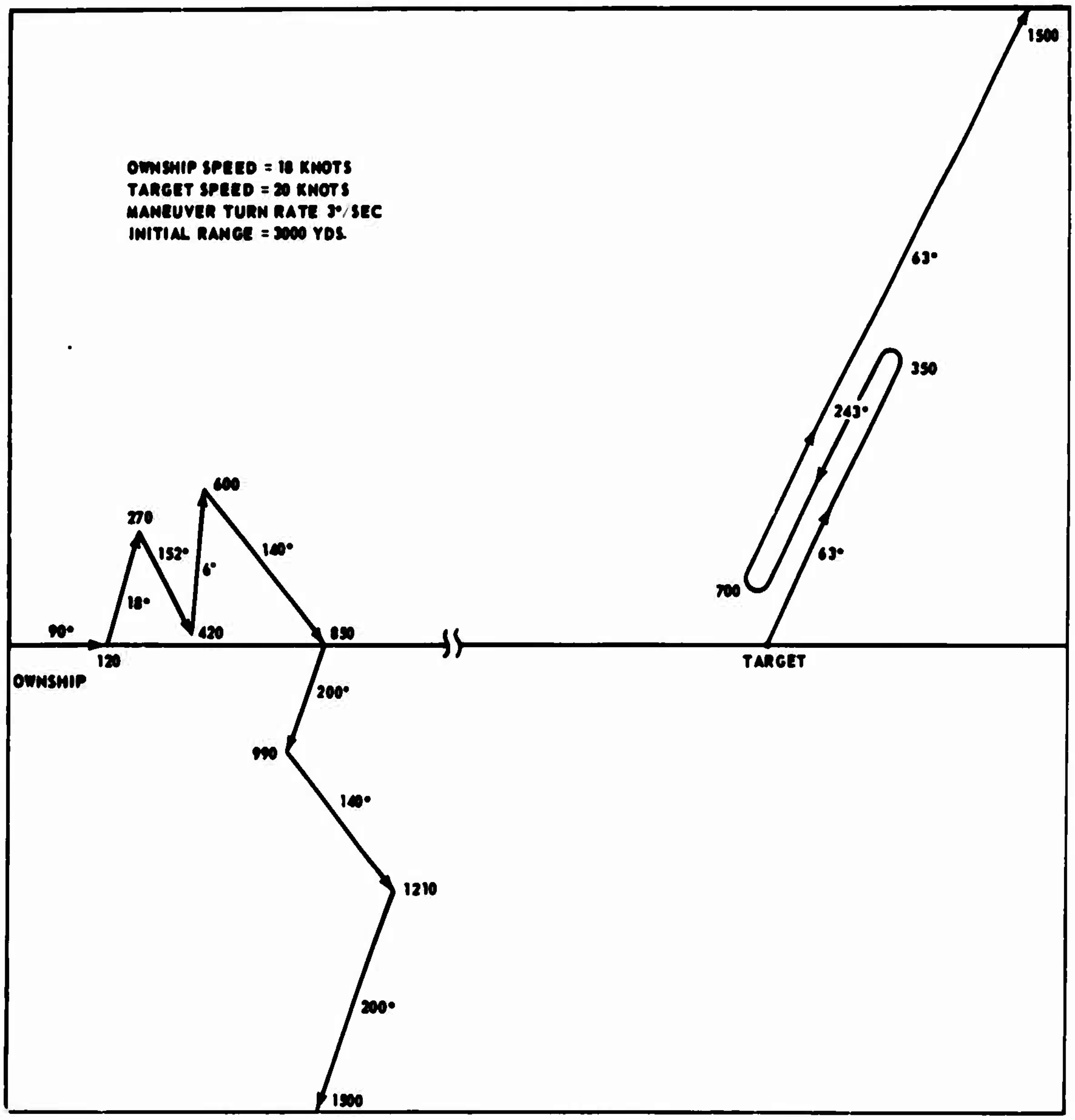

Figure 1. Target-Ownship Geometry

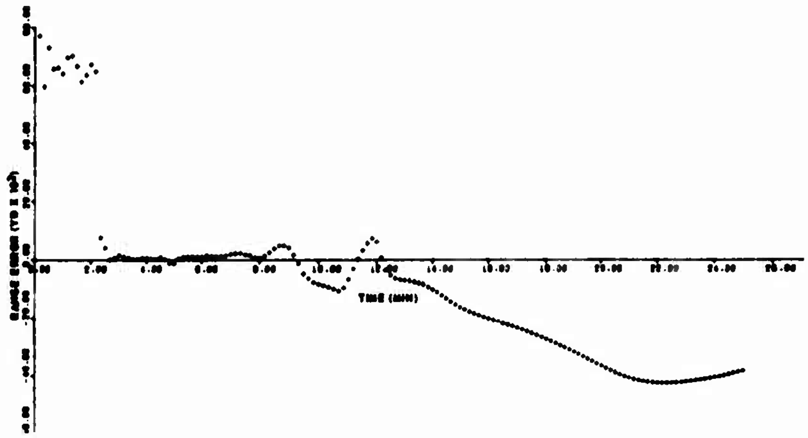

Figure 2. Target Range Error vs Time (Non-Adaptive Kalman Filter)

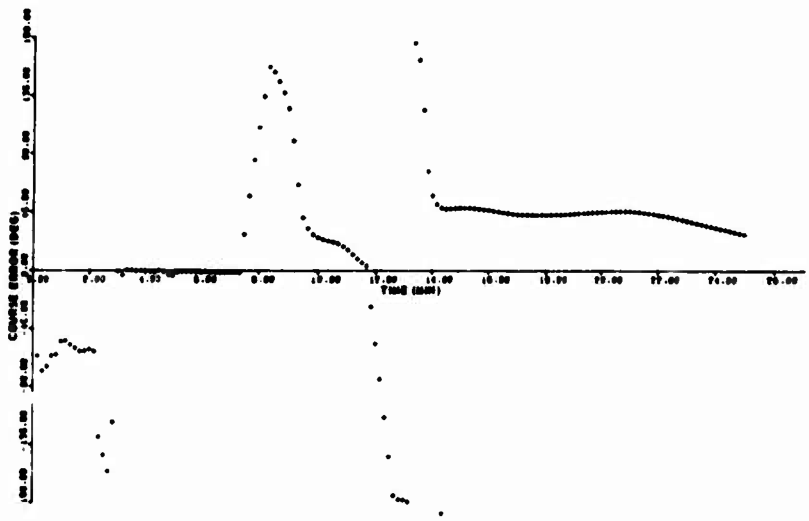

Figure 3. Target Course Error vs Time (Non-Adaptive Kalman Filter)

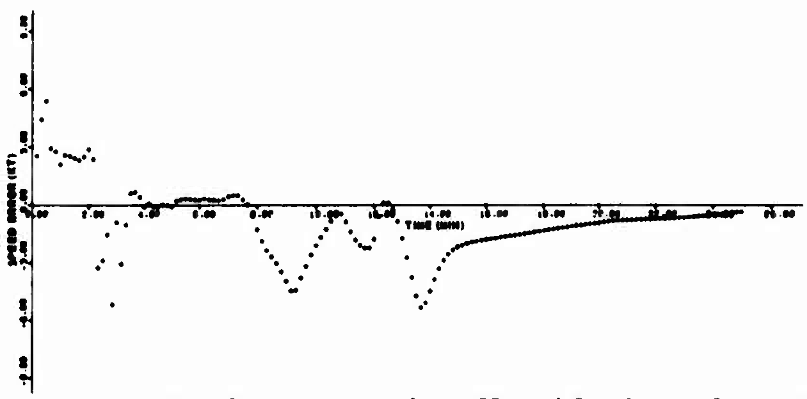

Figure 4. Target Speed Error vs Time (Non-Adaptive Kalman Filter)

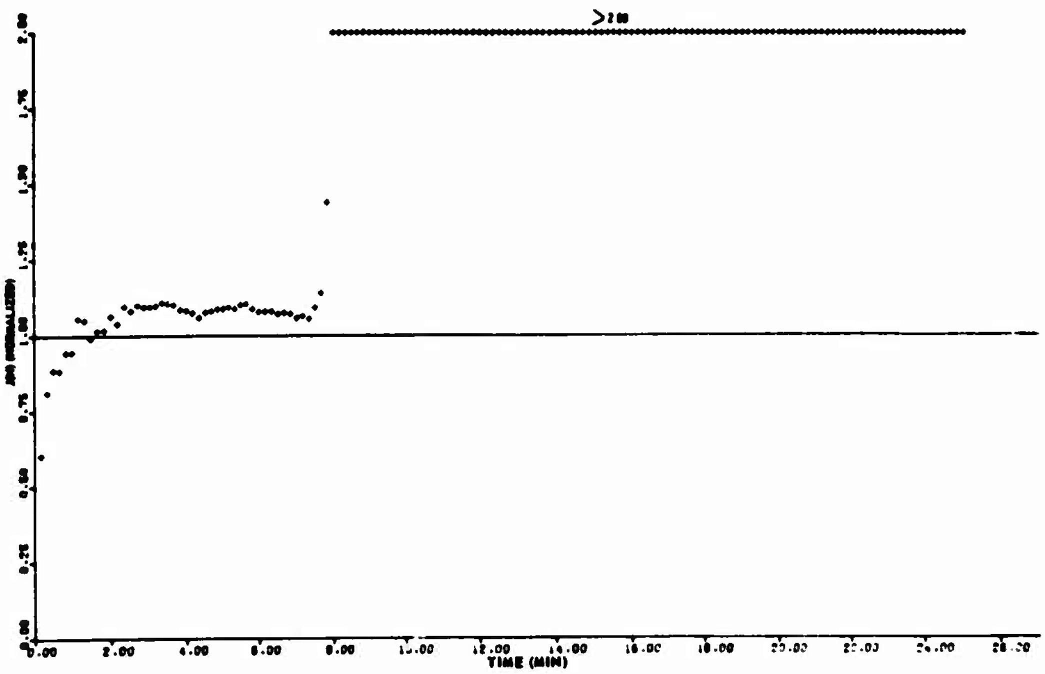

Figure 5. Performance Index vs Time (Non-Adaptive Kalman Filter)

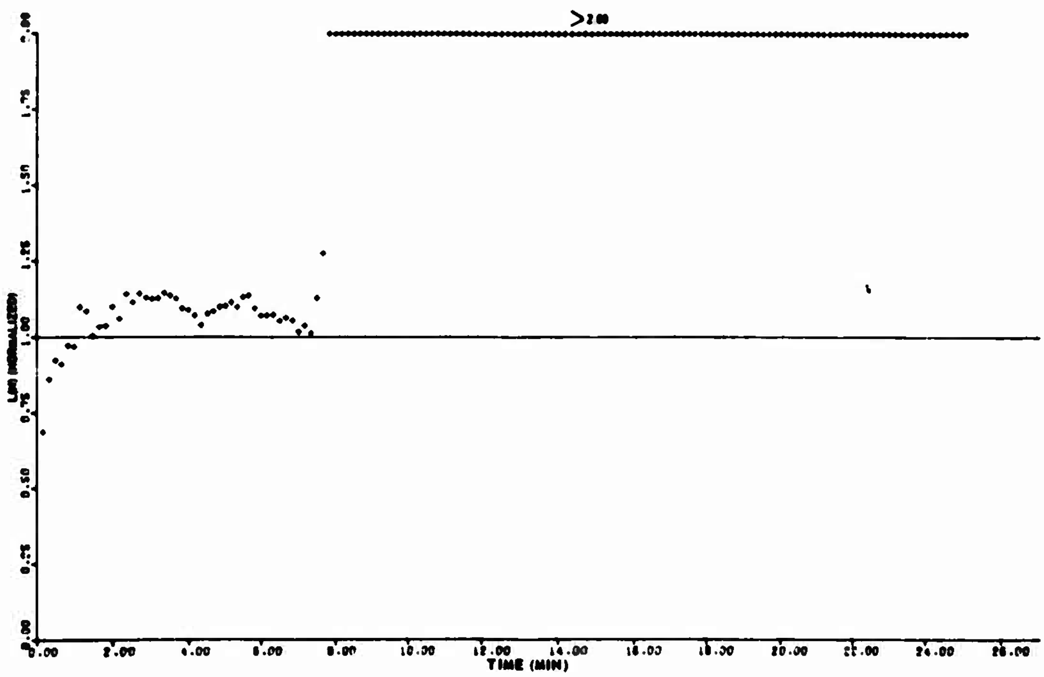

Figure 6. Modified Performance Index vs Time (Non-Adaptive Kalman Filter)

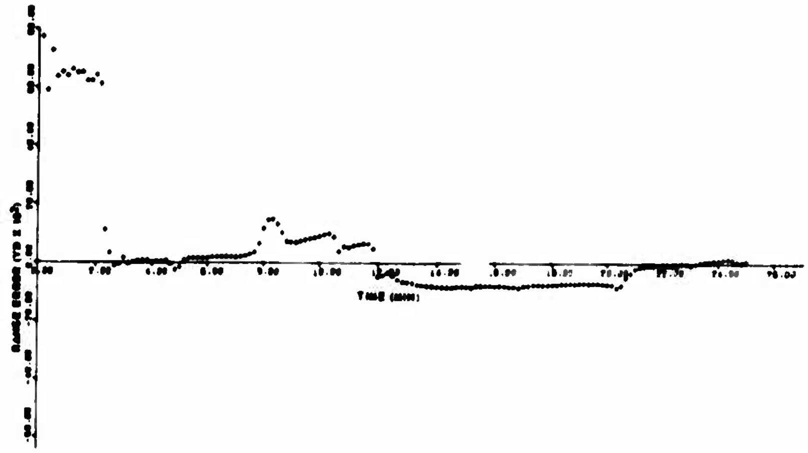

Figure 7. Target Range Error vs Time (Adaptive Kalman Filter)

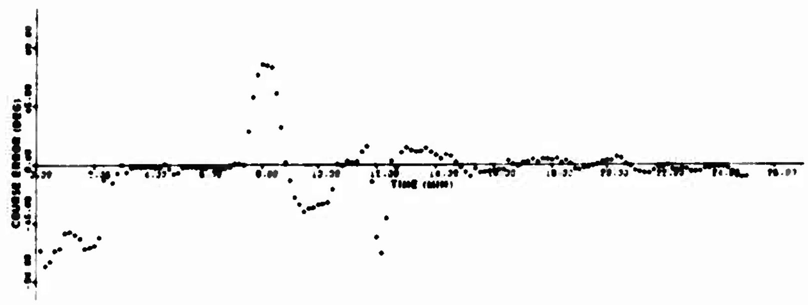

Figure 8. Target Course Error vs Time (Adaptive Kalman Filter)

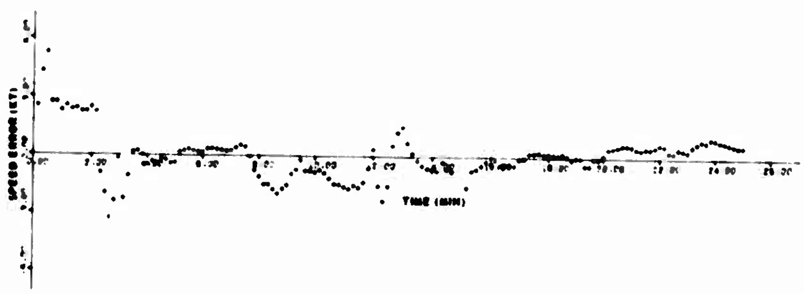

Figure 9. Target Speed Error vs Time (Adaptive Kalman Filter)

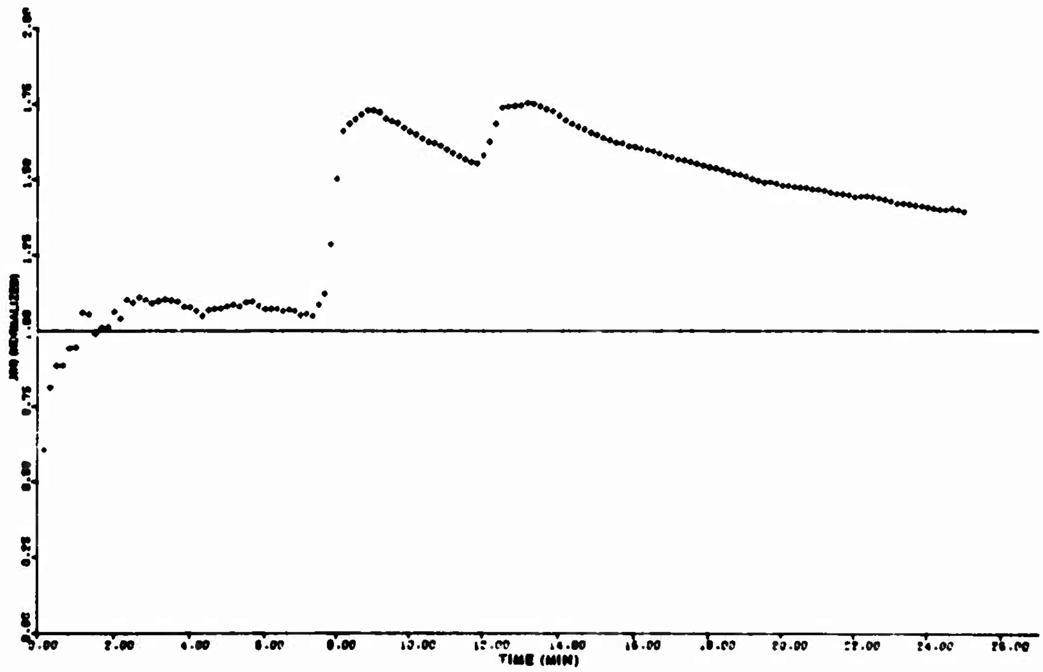

Figure 10. Performance Index vs Time (Adaptive Kalman Filter)

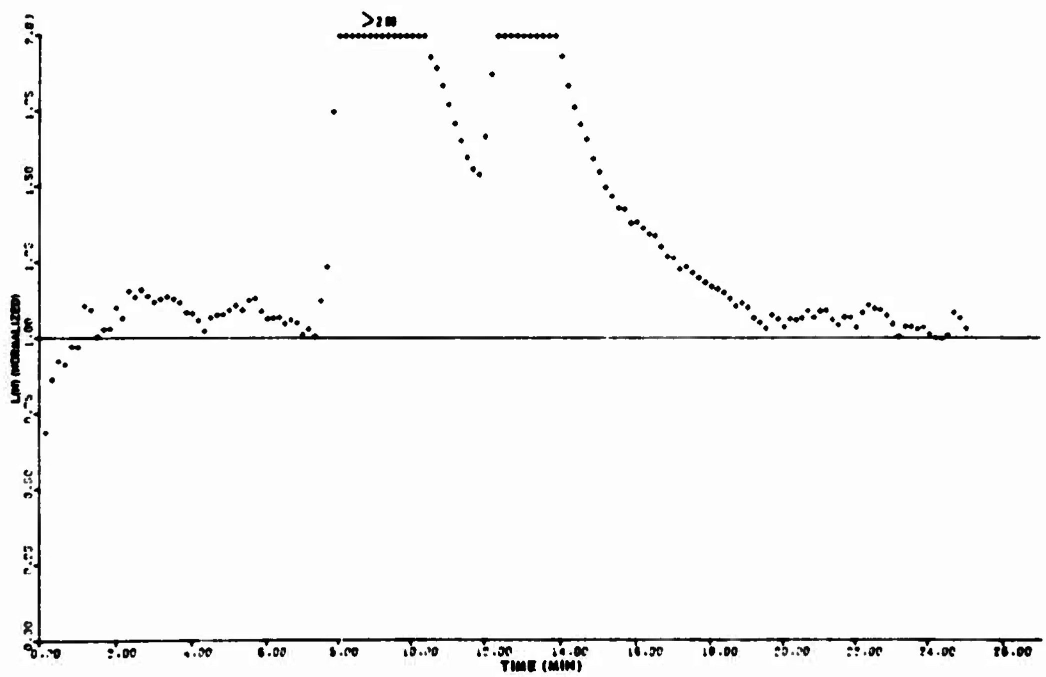

Figure 11. Modified Performance Index vs Time (Adaptive Kalman Filter)

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

An important relationship between the Kalman filter data measurement residuals and the associated system performance index has been established. The computational simplicity of the relationship not only permits on-line evaluation of the performance index, but is further exploited to deduce pertinent statistical properties of the index. These statistics are subsequently utilized to formulate practical adaptive control criteria and to modify the performance index to enhance its reliability as a solution quality indicator. The utility of the modified performance index for assessing filter status and for adaptively regulating the plant noise covariance matrix has been discussed and demonstrated via laboratory simulation.

Even though the simulation results are preliminary they clearly illustrate that a significant improvement in filter performance can be achieved when this type of adaptive control mechanism is appended.

Although the results presented here are encouraging, much work remains to be done. In particular, a quantitative functional relationship between the modified performance index and the plant covariance matrix needs to be established. A study to determine how at-sea data affect the behavior of the modified performance index should also be conducted. Finally, techniques to adaptively compute both the mean and covariance of the plant noise should be further explored. Current efforts are being directed toward the solution of these problems and the results of these investigations will be documented in future reports.

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